RF CAVITY STUDIES AT RHUL
SIMULATIONS & MEASUREMENT

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OUTLINE

• Intro to RF Accelerating Structures
• ACE3P Codes
• Cavity-to-cavity Coupling
  • Cavity Simulations
  • Theoretical model
• HOM coupler
  • Multipacting
• Field Emission
• Bead-pull Test Bench
Try simple azimuthally symmetric trial solution

$$E_z(r, z, t) = R(r)e^{i\omega t}$$

Wave Equation

$$\frac{\partial^2 E_z}{\partial z^2} + \frac{1}{r} \frac{\partial E_z}{\partial r} + \frac{\partial^2 E_z}{\partial r^2} - \frac{1}{c^2} \frac{\partial^2 E_z}{\partial t^2} = 0$$

Boundary Condition: No tangential E field
No normal B field
ACCELERATING MODE

Transverse Magnetic Mode (TM)

\[ E_z = E_0 J_0(k_r r) \cos \omega t \quad B_\theta = -\frac{E_0}{c} J_1(k_r r) \sin \omega t \]

However, not the only mode ...
MODE INDICES - $\text{TM}_{mnp}$ $\text{TE}_{mnp}$

Cylindrical walls allows different radial solutions
• $n^{th}$ root of $m^{th}$ Bessel function

\[
E_z = E_0 J_m(k_m n r) \cos m\phi \cos \frac{p\pi z}{l} e^{i\omega t}
\]

Wrapping of $\phi$ lead to $m$ index
$E(r, 2\pi, z, t) = E(r, 0, z, t)$

End cap boundaries lead to $p$ index
MONOPOLES

TM\textsubscript{01p}

TM\textsubscript{02p}

TM\textsubscript{03p}
DIPOLES

TM_{12p}

TM_{11p}

TM_{13p}
QUADRUPOLES & BEYOND

- TM_{31p}
- TM_{21p}
- TM_{41p}
SCRF ELLIPTICAL CAVITIES

Real cavities aren’t as simple as a pillbox

Parameters chosen to:
Maximise energy transfer to the beam
Minimise chance of breakdown
Match a velocity range

Multi-cell means there are now multiple ways for the modes to occur
COUPLED OSCILLATORS

Eigenmodes of coupled oscillators split according to the phase difference
‘0’-mode, ‘\(\pi\)’-mode, etc.

For \(N+1\) coupled oscillators
\(i\pi/N\) radians phase advance \((i=0,1,2,...N)\)

Frequency also splits
Dependant of coupling strength
Each new mode may be plotted on a Brillouin curve
For \(N < \infty\) the modes are equally spaced along the curve

Dispersion Relation
\[
\omega_\theta^2 = \omega_\pi^2 \left( 1 - k \cos \theta \right)
\]

Coupling
\[
k = \frac{\omega_\pi^2 - \omega_0^2}{\omega_\pi^2 + \omega_0^2}
\]
Wake field can be expanded as multipole series of HOMs

Monopoles can affect acceleration

Dipoles lead to position dependent kicks
  • TM modes cause longitudinal kicks
  • TE modes cause transverse kicks

Affect beam quality
R/Q

Provides a “measure” of the energy exchange between the beam and a mode
Function of cavity geometry

Beta dependence particularly important for proton linacs
ACE3P CODES

• ACE3P suite of parallel electromagnetic codes based on higher-order finite elements

• Omega3P, T3P, S3P, Track3P, TEM3P, Pic3P
  • Simulations ran on Hopper at NERSC
    • 153,216 cpu’s, 217 TB of memory
    • peak performance of 1.28 Petaflops/sec
    • 5th fastest in the world!
CAVITY-TO-CAVITY COUPLING
MULTI-CAVITY COUPLING

A single cell has the usual mode spectrum
\[ \text{TM}_{mnp}, \text{TE}_{mnp} \]

Coupled cell (e.g. in a multi-cell cavity)
Modes split into passbands
Each oscillation characterised by a phase advance per cell

Multi-cavity installations (i.e. a cryomodule)
Modes below cutoff so disregarded
But this neglects evanescent coupling!
**BEAM-PIPE CUTOFF**

Mode propagation in the beam pipe

\[ e^{i\sqrt{k^2-k_c^2}z} = e^{i\frac{2\pi}{c}\sqrt{f^2-f_c^2}z} \]

- \( f < f_c \) means

\[ i\frac{2\pi}{c}\sqrt{f^2-f_c^2}z \rightarrow \text{Real} \]

\[ k_c = \frac{p_{nm}}{a} = \frac{2.4048}{0.04} \sim 60 \]

\[ f_c = \frac{ck_c}{2\pi} \sim 2.871\text{GHz} \]

40cm radius
EIGENSOLVE 4 FULL CAVITIES

~6m long

~880k elements
Average volume = 1.96 x 10^{-7} m^3
Min edge length = 2mm
Max edge length = 24mm
Magnetic symmetry plane

CEA-SACLAY SPL Design
EIGENMODES

Finding the first 100 modes of a four cavity sim uses ~2000 cpu hrs

Each cavity mode will be found four times
one for each cavity

A single cavity will dominate each mode
however the evanescent field allows coupling
THREE GEOMETRIES

- Extended 12cm
- Nominal 6cm
- Notaper

\[ f_c = 1.76 \text{GHz} \]

\[ f_c = 2.87 \text{GHz} \]
$$k = \frac{\omega_\pi^2 - \omega_0^2}{\omega_\pi^2 + \omega_0^2}$$
Simplified Model

Oscillation inside cavity

Decays exponentially inside beam pipe
FINITE POTENTIAL WELL

\[ \psi_j = A_j e^{ik_j z} + B_j e^{-ik_j z} \]

\[ k = \frac{\sqrt{2m_j (V - E)}}{\hbar^2} \]
FINITE POTENTIAL WELL

\[ \psi, \frac{d\psi}{dz} \] must be continuous at each boundary.

Rewrite in terms of matrices

\[ m M_j = \begin{pmatrix} e^{ik_j zm} & e^{-ik_j zm} \\ ik_j e^{ik_j zm} & -ik_j e^{-ik_j zm} \end{pmatrix} \]

Therefore at each boundary

\[ j M_j \begin{pmatrix} A_j \\ B_j \end{pmatrix} = j M_{j+1} \begin{pmatrix} A_{j+1} \\ B_{j+1} \end{pmatrix} \]
At boundary I
\[ 0 \cdot M_0 \begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = 0 \cdot M_1 \begin{pmatrix} A_1 \\ B_1 \end{pmatrix} \]

Therefore
\[ \left[ (1 \cdot M_2)^{-1} \cdot 1 \cdot M_1 \cdot (0 \cdot M_1)^{-1} \cdot 0 \cdot M_0 \right] \begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = \begin{pmatrix} A_2 \\ B_2 \end{pmatrix} \]

Need to find bound state!
Therefore, set \( A_0 = 0 \) and \( B_0 = 1 \)
No backward wave in first region

Solve to find where \( B_2 = 0 \)
No forward wave in last region
N COUPLED WELLS

For N coupled wells

\[
\left( \prod_{2N-1}^{0} [(j M_{j+1})^{-1} * j M_j] \right) \begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = \begin{pmatrix} A_{2N} \\ B_{2N} \end{pmatrix}
\]

Again, solve for \( B_{2N} = 0 \) if \( A_0 = 0, B_0 = 1 \)
DISCRETE ENERGY LEVELS

![Graph showing discrete energy levels with different well configurations](graph.png)
POTENTIAL WELL TO CAVITY

\[ k = \left[ \left( \frac{\omega}{c} \right)^2 - \left( \frac{p_{nm}}{a} \right)^2 \right]^{1/2} \]

Does \( k \) need to change depending on phase advance?

To create cavity, set up a well where the lowest eigenvalue is the resonant frequency using

\[ z = \frac{2 \tan^{-1} \left( \frac{K_1}{K_0} \right)}{K_0} \]
COMPARISON

Designs from R Calaga
COUPLING SUMMARY

Cavity-to-cavity simulations
Determine the degree of coupling
Optimise interconnect region, is a taper necessary?

Calculations using simplified model
Preliminary results show rough agreement for dipole
Can model be improved?
  Change \( k \) according the phase advance?
Are we severely limited by only 1 dimension?
What about modes above cut-off?
MULTIPACTING
HOM COUPLER

Based on tesla style coupler

Scaled to 704.4MHz

Design from R Calaga
MULTIPACTING

Resonant process which lead to electron avalanche

- Absorb RF power
- Heating effects

Electron impacts on surface

- if $\delta > 1$, secondary $e^-$ emitted
- $\mathbf{E}$ points towards surface

$$N_e = N_0 \prod_{m=1}^{k} \delta(K_m)$$

When happens, multipacting is barrier in rising the accelerating field in cavities and usually leads to quench.

In the design process we need to prove whether or not the shape of cell allows for multipacting.

SEY is function of the impact energy $K$ and depends on the surface cleanness.
ONE-POINT MULTIPACTING

$e^-$ impact very close to emission site
- Primarily accelerated by $E_{\perp}$
- Surface $H$ forces $e^-$ along quasi-cyclotron orbits

**MP will occur in region where $H$ does not vary much along the cavity wall**

Hence elliptical cavities
- Varying magnetic field
- $E_{\perp}$ tiny at equator
TWO-POINT MULTIPACTING

Resonance typically occurs when time between impacts is half-integer multiple of RF period

Common in couplers due to small gaps between surfaces
MULTIPACTING

3MV/m
MULTIPACTING REGIONS

Graphs showing resonant energy and electric field magnitude against field gradient [MV/m].

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ALTERNATE DESIGN

Designs from H-W Glock
MP SUMMARY

• Future
  • Perform positional & rotational studies
  • Modify geometry
    • Add grooves to inside of outer can?
  • Use transient fields measure effects of HOMs
• Determine the best design
FIELD EMISSION

Fowler-Nordheim Law

\[
J(r, t) = 1.54 \times 10^{(-6 + \frac{4.52}{\sqrt{\phi}})} \left( \frac{\beta E}{\phi} \right)^2 e\left( \frac{-6.53 \times 10^9 \phi^{1.5}}{\beta E} \right)
\]
FIELD EMISSION GOALS

Determine heat load produced

Determine if there is any affect on the beam

Determine if taper is required to trap the dark current
BEAD-PULL
Slater’s Theorem

\[
\frac{\Delta \omega_0}{\omega_0} = \frac{\int_{\Delta V} (\mu_0 H^2 - \epsilon_0 E^2) dV}{\int_V (\mu_0 H^2 + \epsilon_0 E^2) dV}
\]

If a small bead is inserted into the cavity, the perturbation shifts the resonant frequency

\[
\frac{\Delta \omega_0}{\omega_0} = -\frac{3 \Delta V}{4U} \left( \frac{\varepsilon_r - 1}{\varepsilon_r + 2} \epsilon_0 E^2 + \frac{\mu_r - 1}{\mu_r + 2} \mu_0 H^2 \right)
\]
For a spherical dielectric bead, we choose $\mu_r = 1$

$$\frac{\Delta \omega_0}{\omega_0} = -\frac{3\Delta V}{4U} \frac{\varepsilon_r - 1}{\varepsilon_r + 2} \varepsilon_0 E^2$$

Scan along 5 dof possible $x, y, z, x', y'$
RFQ PROTOTYPE FOR FETS

M1 319.725MHz
M2 328.275MHz

S11 Magnitude [dB]

Frequency [Mhz]
SCANS

319.725MHz

Probably off centre

Dipole

328.275MHz

Quadrupole
BEAD-PULL TO DO

Introduce a read back system
  Motors may miss steps
  Thread may slip

Increase mechanical stability
SUMMARY

Cavity-to-cavity coupling calculations
Cpu-heavy sims to try to understand coupling
Help optimise interconnect regions, is a taper necessary
Simplified model being developed

Preliminary simulations of Multipacting in coupler
Many more studies needed but initial results suggest modifications may be necessary for tesla type coupler

Field Emission
Future studies to determine heat load
Is a taper necessary?

Bead-pull up and running
read-back to be installed to provide accuracy and reproducibility